## **The Related Samples T Test**

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| --- | --- | --- |
| **t-Test: Paired Two Sample for Means** | | |
|  |  |  |
|  | ***Con1*** | ***Con2*** |
| Mean | 172.6 | 159.4 |
| Variance | 750.2666667 | 789.3777778 |
| Observations | 10 | 10 |
| Pearson Correlation | 0.863335004 |  |
| Hypothesised Mean Difference | 0 |  |
| df | 9 |  |
| t Stat | 2.874702125 |  |
| P(T<=t) one-tail | 0.009167817 |  |
| t Critical one-tail | 1.833112933 |  |
| P(T<=t) two-tail | 0.018335635 |  |
| t Critical two-tail | 2.262157163 |  |
| Difference in Mean | 13.2 |  |

**Data**:



 The situation is analogous:

* They first did a **two-tailed** test to see if the mean impurity differs between two filtration agents.
* Now they want a **one-tailed** test to see if **Filter Agent 1 is more effective** (meaning: lower impurity for Agent 1 → but careful: more effective means **lower impurity**, so if impurity is the measured variable, then  is the alternative hypothesis if  = impurity level).

But in our container example, **Con1** had **higher** sales → better.  
So in the filter case, we need to know: is “more effective” lower impurity or higher sales?

From the phrasing:

“one-tailed test … to determine whether Filter Agent 1 was the more effective”

More effective means **lower impurity** (since impurity is bad). So:

* Let  = population mean impurity for Agent 1
* Let  = population mean impurity for Agent 2

**Hypotheses**:

**Mapping to the container example**

In the container example, **higher** is better (sales).  
In the filter example, **lower** is better (impurity).

So the roles of “which is better” are reversed in terms of the sign of the difference.

In the container data:  
Sample mean difference (Con1 − Con2) = **+13.2** (positive).

If we simply **swap the roles** to match the filter scenario:

Suppose in the actual filter data, the sample mean difference (Agent1 − Agent2) for impurity is **−13.2** (negative) — that would mean Agent1 has lower impurity.

Then the t-statistic would be **−2.8747** instead of +2.8747.

**One-tailed p-value from the given output**

From the container output:

* **One-tailed p-value** = 0.009167817 — this is for the alternative hypothesis  (since t = +2.8747).

If instead we test  with t = +2.8747, the one-tailed p-value would be , which is **not significant**.

But in the filter case, if Agent1 is more effective, we expect **t = −2.8747** (if the data difference in impurity is like the negative of the container sales difference).

Then the one-tailed p-value for  would be the left-tail area for t = −2.8747, which equals **0.009167817** (same as the right-tail area for +2.8747 by symmetry).

**Conclusion for the filter problem**

If the actual two-tailed test in the filter data gave a two-tailed p-value of 0.0183 (like here), then:

* For the one-tailed test in the correct direction (Agent1 better → lower impurity), the p-value is **0.00917**.
* This is significant at  (and even at ).

So we **reject H0** and conclude **Filter Agent 1 is indeed more effective**.

Therefore,

***Significant evidence that Filter Agent 1 is more effective***